5-8 Areas of Circles, Sectors, and Segments of Circles

What You'll Learn

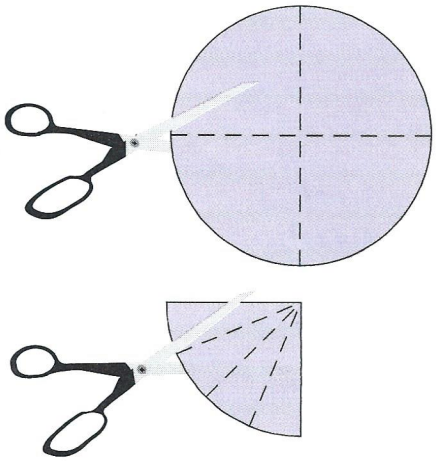
- Computing the areas of circles, sectors, and segments of circles

...And Why

To solve real-world problems in food preparation, archaeology, and biology

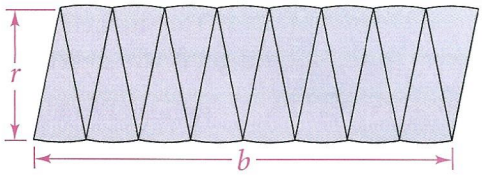
What You'll Need

- compass
- scissors
- tape



WORK TOGETHER

Work in groups. Have each member of your group use a compass to draw a large circle. Fold the circle in half horizontally and vertically. Cut the circle into four wedges on the fold lines. Then fold each wedge into quarters. Cut each wedge on the fold lines. You will have 16 wedges. Tape the wedges to a piece of paper to form the figure below.



1. How does the area of the figure compare with the area of the circle?
2. The base of the figure is formed by arcs of the circle. Write an equation relating the length of the base b to the circumference C of the circle.
3. Write an equation for the length of the base b in terms of the radius r of the circle.
4. If you increase the number of wedges, the figure you create becomes more and more like a rectangle with base b and height r. Write an expression for the area of the rectangle in terms of r.

THINK AND DISCUSS

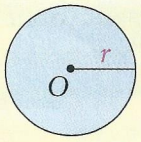
Areas of Circles

Your observations in the Work Together suggest the following theorem.

Theorem 5-12
Area of a Circle

The area of a circle is the product of π and the square of the radius.

$$A = \pi r^2$$



5. **Try This** What is the area of a circle with radius 15 cm? Leave your answer in terms of π.

Example 1 **Relating to the Real World**

Food The diameter of a small pizza is 10 in. How much more pizza do you get if you order a medium pizza with diameter 12 in.?

$$\text{radius of small pizza} = \frac{10}{2} = 5 \qquad r = \frac{d}{2}$$

$$\text{radius of medium pizza} = \frac{12}{2} = 6 \qquad r = \frac{d}{2}$$

Use the formula for the area of a circle.

$$\text{area of small pizza} = \pi(5)^2 = 25\pi \qquad A = \pi r^2$$

$$\text{area of medium pizza} = \pi(6)^2 = 36\pi \qquad A = \pi r^2$$

$$\text{difference in area} = 36\pi - 25\pi = 11\pi$$

11 × π = 34.557519

The medium pizza has about 35 in.^{2} more pizza than the small pizza.

6. Suppose the small pizza in Example 1 costs \$5.00 and the medium pizza costs \$6.00. Which pizza is a better buy? Explain your answer.

Sectors and Segments

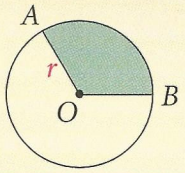
A **sector of a circle** is the region bounded by two radii and their intercepted arc. A slice of pizza is an example of a sector of a circle. You name a sector using one endpoint of the arc, the center of the circle, and the other endpoint of the arc. Sector XOY is at the left.

The area of a sector is a fractional part of the area of a circle. The ratio of a sector's area to a circle's area is $\frac{\text{measure of the arc}}{360}$.

Theorem 5-13
Area of a Sector of a Circle

The area of a sector of a circle is the product of the ratio $\frac{\text{measure of the arc}}{360}$ and the area of the circle.

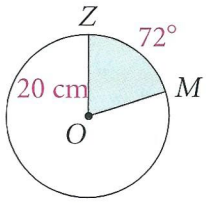
Area of sector $AOB = \frac{m\overset{\frown}{AB}}{360} \cdot \pi r^2$



Example 2

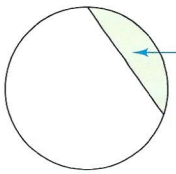
Find the area of sector ZOM. Leave your answer in terms of π.

$$\text{area of sector } ZOM = \frac{m\overset{\frown}{ZM}}{360} \cdot \pi r^2$$
$$= \frac{72}{360} \cdot \pi(20)^2$$
$$= 80\pi$$

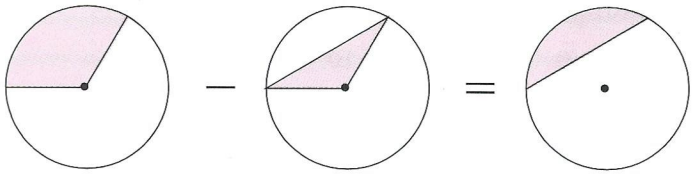


The area of sector ZOM is 80π cm^2.

7. Calculator A circle has diameter 8.2 m. What is the area of a sector with a 125° arc? Round your answer to the nearest tenth.

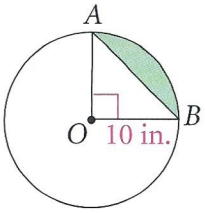


The part of a circle bounded by an arc and the segment joining its endpoints is a **segment of the circle.** To find the area of a segment, draw radii to form a sector. The area of the segment equals the area of the sector minus the area of the triangle formed.



Area of sector — Area of triangle — Area of segment

Example 3



Find the area of the shaded segment.

$$\text{area of sector } AOB = \frac{m\overset{\frown}{AB}}{360} \cdot \pi r^2 \qquad \text{Formula for area of a sector}$$
$$= \frac{90}{360} \cdot \pi(10)^2 \qquad \text{Substitute.}$$
$$= \frac{1}{4} \cdot 100\pi$$
$$= 25\pi$$

$$\text{area of } \triangle AOB = \frac{1}{2}bh \qquad \text{Formula for area of a triangle}$$
$$= \frac{1}{2}(10)(10) \qquad \text{Substitute.}$$
$$= 50$$

$$\text{area of segment} = 25\pi - 50$$

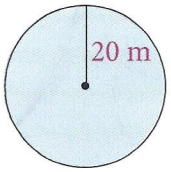
25 × π − 50 = 28.539816

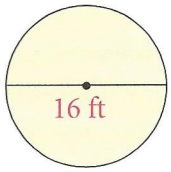
The area of the segment is about 28.5 in.^{2}

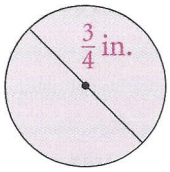
8. Try This A circle has radius 12 cm. Find the area of a segment of the circle bounded by a 60° arc and the segment joining its endpoints. Round your answer to the nearest tenth.

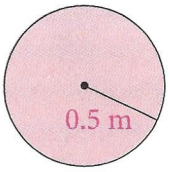
Exercises ON YOUR OWN

Find the area of each circle. Leave your answers in terms of π.

1. 

2. 

3. 

4. 

5. A circle has area 225π m^2. What is its diameter?

6. How many circles with radius 4 in. will have the same total area as a circle with radius 12 in.?

7. **Coordinate Geometry** The endpoints of a diameter of $\odot A$ are (2, 1) and (5, 5). Find the area of $\odot A$. Leave your answer in terms of π.

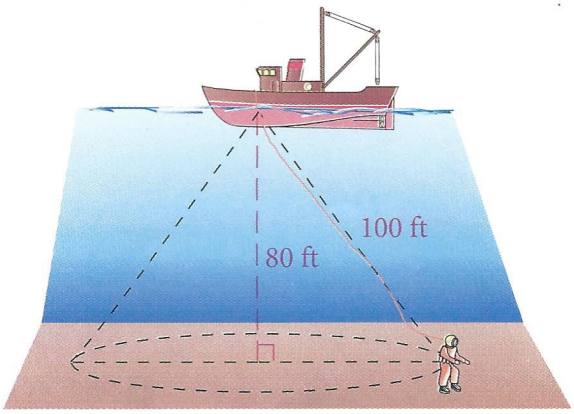
Calculator **Find the area of each circle. Round your answer to the nearest hundredth.**

8. $r = 7$ ft

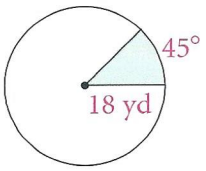
9. $d = 8.3$ m

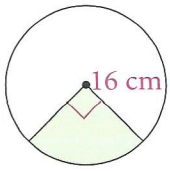
10. $d = 0.24$ cm

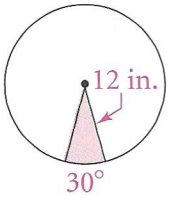
11. **Archaeology** Off the coast of Sweden, divers are working to bring up artifacts from a ship that sank several hundred years ago. The line to a diver is 100 ft long, and the diver is working at a depth of 80 ft. What is the area of the circle that the diver can cover? Round your answer to the nearest square foot.

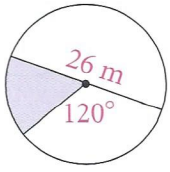


Find the area of each shaded sector of a circle. Leave your answer in terms of π.

12. 

13. 

14. 

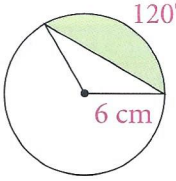
15. 

16. **Games** A dartboard has a diameter of 20 in. and is divided into 20 congruent sectors. Find the area of one sector. Round your answer to the nearest tenth.

17. **Animal Habitats** In the Pacific Northwest, a red fox has a circular home range with a radius of about 718 m. To the nearest thousand, about how many square meters are in a red fox's home range?

Calculator **Find the area of each shaded segment of a circle. Round your answer to the nearest hundredth.**

18.



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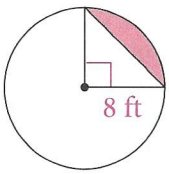

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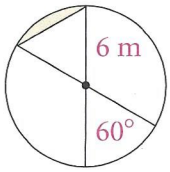

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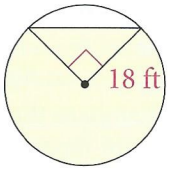
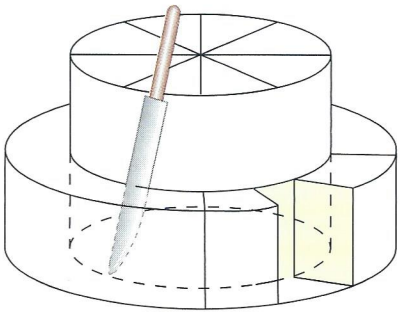


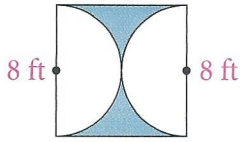
22. **Writing** The American Institute of Baking suggests a technique for cutting and serving a tiered cake. The tiers of a cake have the same height and have diameters 8 in. and 13 in. The top layer and the circle directly under it are cut into 8 pieces and the exterior ring of the 13-inch layer is cut into 12 pieces. Which piece would be biggest, a top, bottom-inside, or bottom-outside piece? Explain your answer.



23. A sector of a circle with a 90° arc has area 36π in.^{2}. What is the radius of the circle?

Find the area of the shaded figure. Leave your answer in terms of π.

24.



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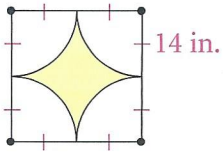

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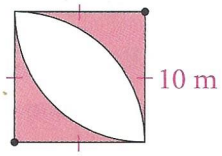

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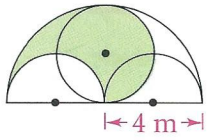

28. **Open-ended** Draw a diagram for a sector of a circle such that the sector has area 16π cm^2. Label the radius of the circle and the measure of the arc of the sector.

29. An 8 ft-by-10 ft floating dock is anchored in the middle of a pond. The bow of a canoe is tied to one corner of the dock with a 10-ft rope.

a. Sketch a diagram of the area in which the bow of the canoe can travel.

b. Write a plan for finding the area.

c. Find the area. Round your answer to the nearest square foot.