6-2 Surface Areas of Prisms and Cylinders

What You'll Learn

- Investigating the surface areas and lateral areas of prisms and cylinders

...And Why

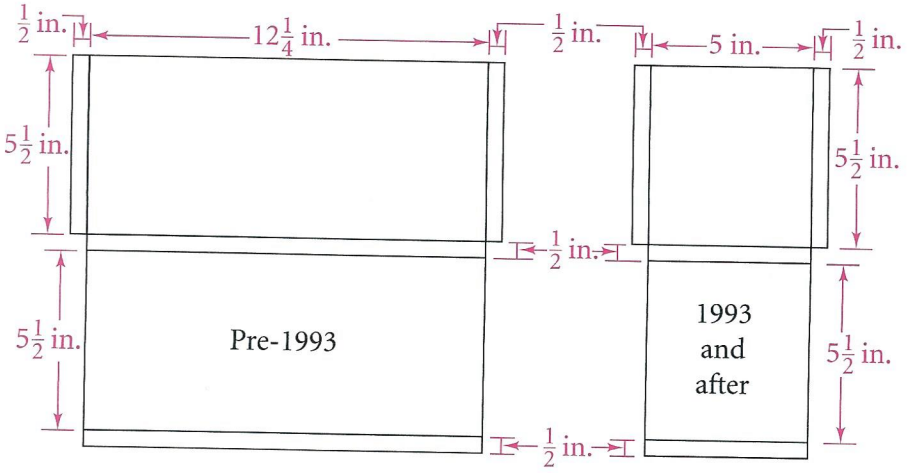
To use surface areas of objects in daily life, from CD cases and videocassette boxes to buildings and storage tanks

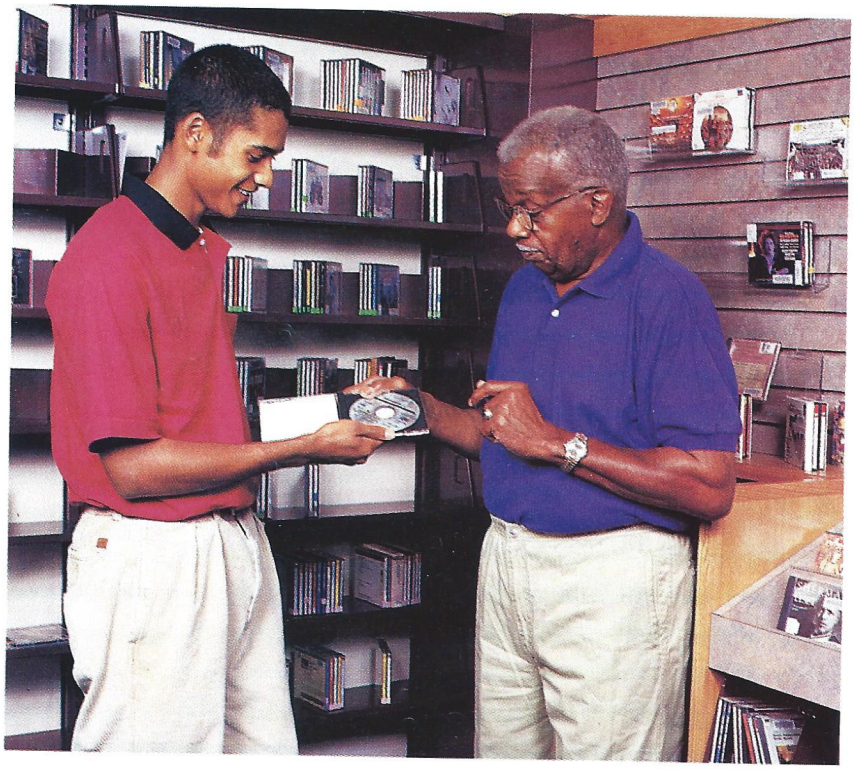
What You'll Need

- straightedge
- $\frac{1}{4}$-in. graph paper

WORK TOGETHER

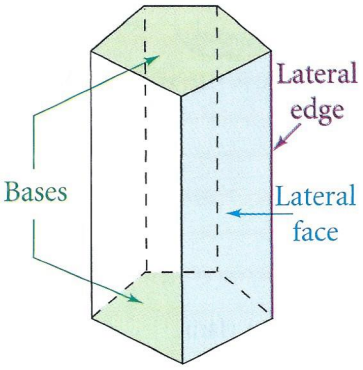
In 1993 the music recording industry changed the size of compact disc packaging. The change to smaller packaging was made in response to the concerns of major recording artists who were worried about the effects of wasted packaging on the environment. Nets for the two packages are shown.





Work with a group. Draw nets on graph paper for each of the two packages.

1. **a.** What is the area of the net for the pre-1993 CD packaging?
   b. What is the area of the net for the new, smaller CD packaging?
   c. How many square inches of packaging are saved by using the smaller packaging?
2. How many pairs of congruent rectangles are in each net?
3. ***Critical Thinking*** Why do you think the earlier packaging was so large?



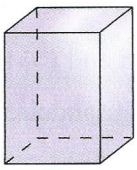
pentagonal prism

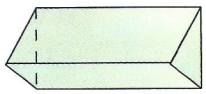
THINK AND DISCUSS

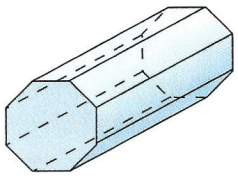
Lateral Areas and Surface Areas of Prisms

The CD packages in the Work Together are examples of rectangular prisms. A **prism** is a polyhedron with two congruent, parallel **bases.** The other faces are **lateral faces.** A prism is named for the shape of its bases.

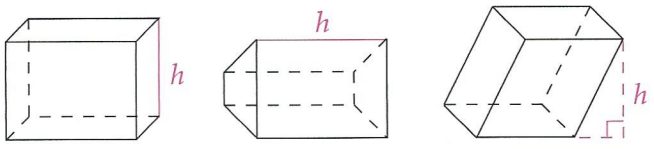
4. Match each prism with one of the following names: triangular prism, rectangular prism, hexagonal prism, octagonal prism.

a. 

b. 

c. 

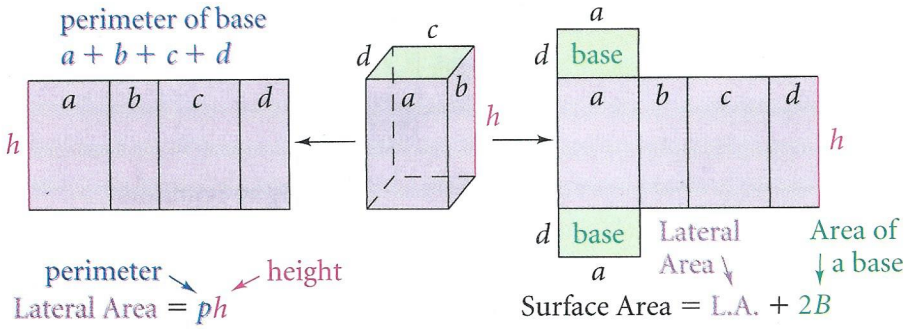
An **altitude** of a prism is a perpendicular segment that joins the planes of the bases. The **height** h of the prism is the length of an altitude. A prism may be either *right* or *oblique.* In a **right prism** the lateral faces are rectangles and a lateral edge is an altitude. In this book you may assume that a prism is a right prism unless you are told otherwise.



right prisms **oblique prism**

5. **Open-ended** Draw a right triangular prism and an oblique triangular prism. Draw and label an altitude in each figure.

The **lateral area** of a prism is the sum of the areas of the lateral faces. The **surface area** is the sum of areas of the lateral faces and the two bases. You can also find these areas by using formulas.



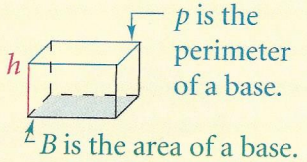
Theorem 6-1
Lateral and Surface Areas of a Right Prism

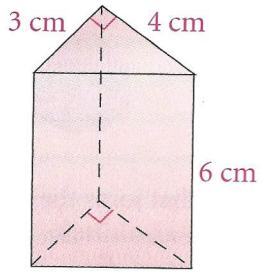
The lateral area of a right prism is the product of the perimeter of the base and the height.

$$\text{L.A.} = ph$$

The surface area of a right prism is the sum of the lateral area and the areas of the two bases.

$$\text{S.A.} = \text{L.A.} + 2B$$





Example 1

Find (a) the lateral area and (b) the surface area of the prism.

The hypotenuse of the triangular base is 5 cm, because the sides form a Pythagorean triple.

a. $\text{L.A.} = ph$
$= 12 \cdot 6$
$= 72$

Use the formula for Lateral Area.
$p = 3 + 4 + 5 = 12$ cm

The lateral area of the prism is 72 cm^2.

b. $\text{S.A.} = \text{L.A.} + 2B$
$= 72 + 2(6)$
$= 84$

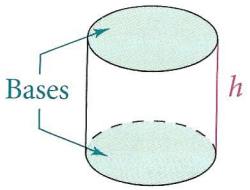
Use the formula for Surface Area.
$B = \frac{1}{2}(3 \cdot 4) = 6$ cm^2

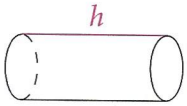
The surface area of the prism is 84 cm^2.

6. Try This A **cube** is a prism with square faces. Suppose a cube has edges 5 in. long. What is its lateral area? its surface area?

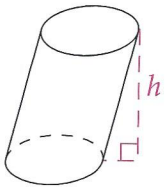
Lateral Areas and Surface Areas of Cylinders

Like a prism, a **cylinder** has two congruent parallel bases. However, the bases of a cylinder are circles. An **altitude** of a cylinder is a perpendicular segment that joins the planes of the bases. The **height** h of a cylinder is the length of an altitude.





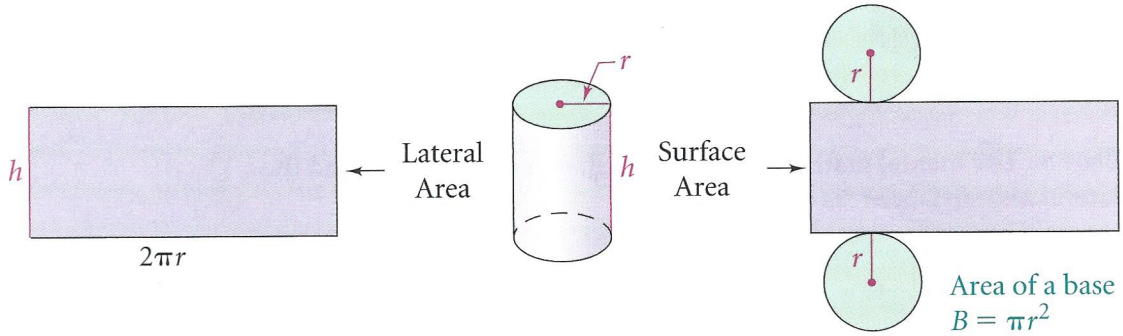
right cylinders



oblique cylinder

In this book you may assume that a cylinder is a *right* cylinder, like the first two cylinders above, unless you are told otherwise.

To find the lateral area of a cylinder, visualize "unrolling" it. The area of the resulting rectangle is the **lateral area** of the cylinder. The **surface area** of a cylinder is the sum of the lateral area and the areas of the two circular bases. You can find formulas for these areas by looking at a net for a cylinder.



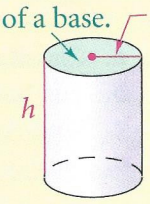
Theorem 6-2
Lateral and Surface Areas of a Right Cylinder

The lateral area of a right cylinder is the product of the circumference of the base and the height of the cylinder.

$$\text{L.A.} = 2\pi rh, \text{ or L.A.} = \pi dh$$

The surface area of a right cylinder is the sum of the lateral area and the areas of the two bases.

$$\text{S.A.} = \text{L.A.} + 2B$$



7. Try This The radius of the base of a cylinder is 4 in. and its height is 6 in.

a. Find the lateral area of the cylinder in terms of π.

b. Find the surface area of the cylinder in terms of π.

Example 2 **Relating to the Real World**

Machinery The wheel of the steamroller at the left is a cylinder. How many square feet does a single revolution of the wheel cover? Round your answer to the nearest square foot.

The area covered is the lateral area of a cylinder that has a diameter of 5 ft and a height of 7.2 ft.

$$\begin{aligned} \text{L.A.} &= \pi dh && \text{Use the formula for Lateral Area of a cylinder.} \\ &= \pi(5)(7.2) && \text{Substitute.} \\ &= 36\pi && \text{Simplify.} \end{aligned}$$

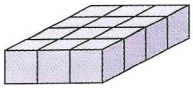
36 × π = 113.09734

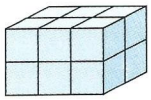
A single revolution of this steamroller wheel covers about 113 ft^2.

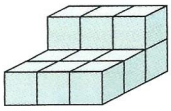
8. Estimation Use $\pi \approx 3$ to estimate the lateral area and surface area of a cylinder with height 10 cm and radius 10 cm.

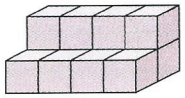
Exercises ON YOUR OWN

Each structure is made of 12 unit cubes. What is the surface area of each figure?

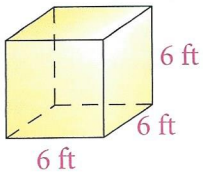
1. 

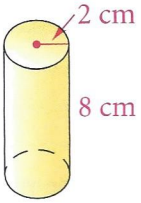
2. 

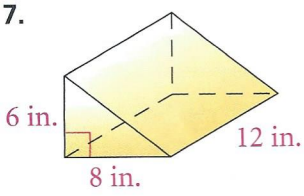
3. 

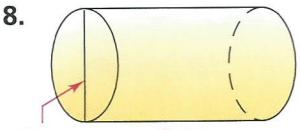
4. 

Choose **Use mental math, paper and pencil, or a calculator to find the lateral and surface areas of each figure.**

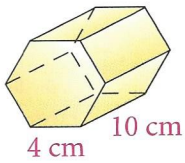
5. 

6. 

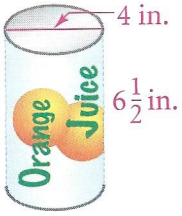
7. 

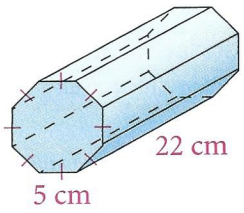
8. 

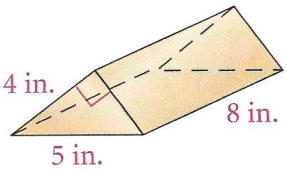
9. **a.** Classify the prism.
   b. The bases are regular hexagons. Find the sum of their areas.
   c. Find the lateral area of the prism.
   d. Find the surface area of the prism.

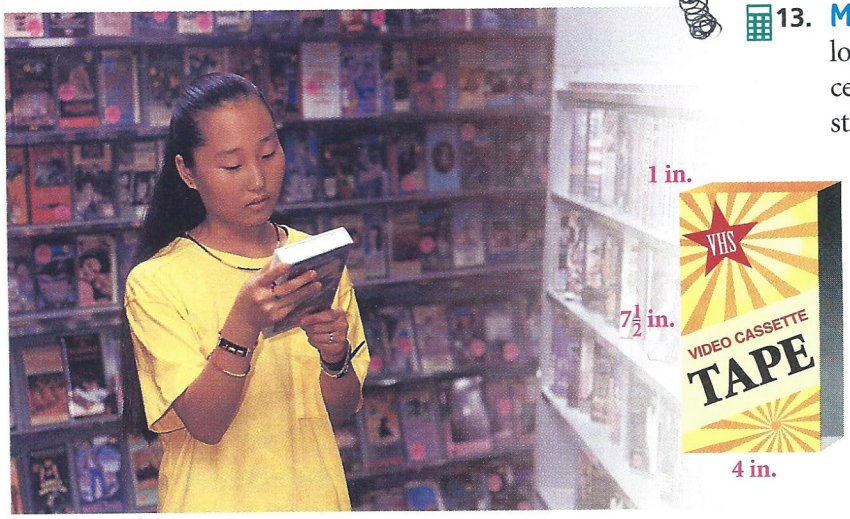


Calculator **Find the lateral area of each object. When an answer is not a whole number, round to the nearest tenth.**

10. 

11. 

12. 



13. **Manufacturing** A standard drinking straw is 195 mm long and has a diameter of 6 mm. How many square centimeters of plastic are needed to make 1000 straws? Round your answer to the nearest hundred.

14. **Packaging** A typical video cassette tape box is open on one side. How many square inches of cardboard are in a typical video-cassette tape box?

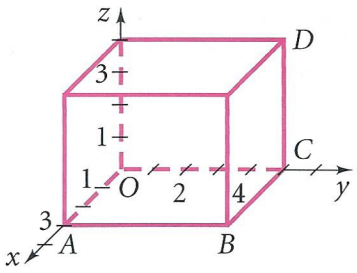
15. **Algebra** A triangular prism has base edges 4 cm, 5 cm, and 6 cm long. Its lateral area is 300 cm^2. What is the height of the prism?

16. **Open-ended** Draw a net for a rectangular prism with a surface area of 220 cm^2.

17. a. Geometry in 3 Dimensions List the three coordinates (x, y, z) for vertices A, B, C, and D of the rectangular prism.

b. What is AB? **c.** What is BC? **d.** What is CD?

e. What is the surface area of the prism?

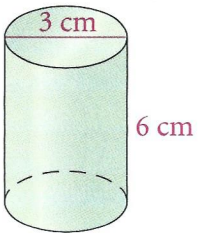


18. Estimation Estimate the surface area of a cube with edges 4.95 cm long.

19. Writing Explain how a cylinder and a prism are alike and how they are different.

Calculator Find the surface area of each figure. When an answer is not a whole number, round to the nearest tenth.

20.



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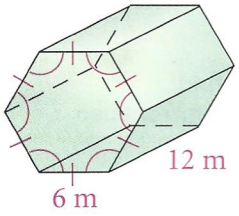

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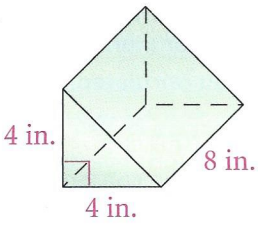

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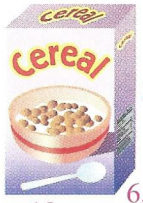
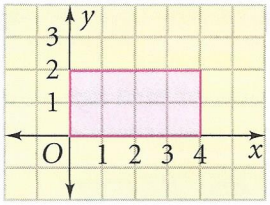


24. a. Coordinate Geometry Suppose the rectangle shown at the right is rotated 360° about the y-axis. What space figure will the rotating rectangle generate?

b. Find the surface area of this figure in terms of π.

c. What will be the surface area in terms of π if the rectangle is rotated 360° about the x-axis?



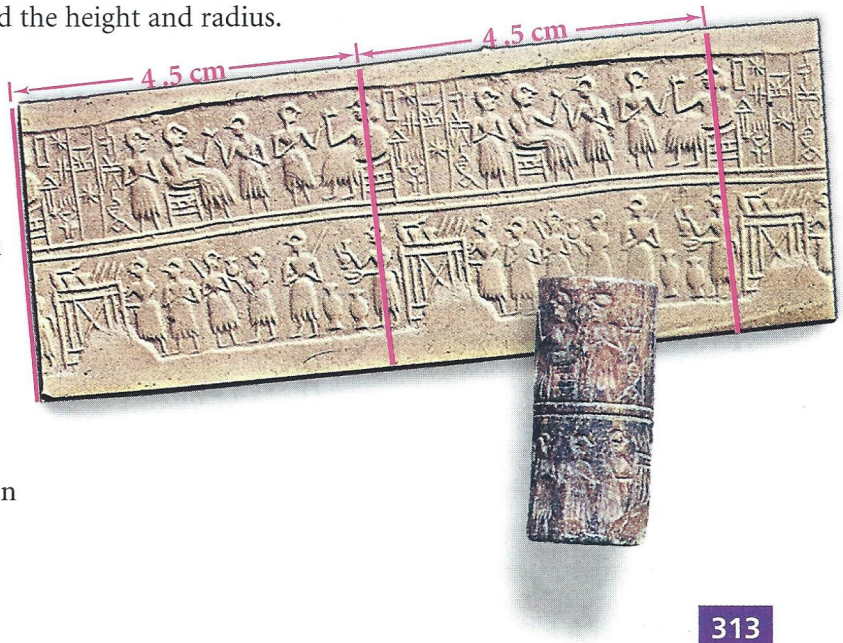
25. Standardized Test Prep If the radius and height of a cylinder are both doubled, then the surface area is ■.

A. the same **B.** doubled **C.** tripled **D.** quadrupled

E. not enough information given to determine the amount of change in the surface area

26. Algebra The sum of the height and radius of a cylinder is 9 m. The surface area of the cylinder is 54π m^2. Find the height and radius.

27. Frieze Patterns From about 3500 B.C. to 2500 B.C., Sumerians etched cylindrical stones to form seals. They used the imprint from rotating the seal to make an official signature. These seals make interesting frieze patterns.



a. Two and one-quarter revolutions of a cylinder created the frieze pattern at the right. What are the dimensions of the cylinder? Round to the nearest tenth.

b. What type of transformation appears in the frieze pattern?